RETURN CALCULATIONS

We denote by:
\( C_t \): Cash amount at time \( t \),
\( W_t \): Withdrawals at time \( t \),
\( D_t \): Deposits at time \( t \).

Return Numerator

The numerator is equal to: \( C - C_{t-1} + W_t - D_t \)

Return Denominator

There are two methods for defining the base in the return calculation:

Method 1 (Net Liquidation Value). The Net Liquidation Value (NLV) is the value of the account—i.e., the cash value of the account if all open positions were liquidated at their settlement values. The NLV on the previous day’s settlement is used as the base for calculating the return. This approach is straightforward and simple and works well for fully funded accounts. The problem, however, is that it can lead to distorted returns and volatility for margined accounts (both values will be too high). NLV is often completely inappropriate for calculating returns for futures and FX accounts, which can be heavily margined.

Method 2 (Nominal Account Size). The NAS is the constant nominal account size defined by the trader, which can only be higher than the NLV. The trader specifies the Nominal Account Size (NAS) that should be used as the base for calculating returns. The NAS specified by the trader is adjusted on subsequent days to reflect daily profit/loss. For example, if a trader with an NLV of $100,000 specifies an NAS of $500,000 and then has cumulative profits of $50,000, the NAS would then be $550,000. Deposits and withdrawals do NOT affect the NAS. Traders can redefine the NAS at any time. Such redefined NAS levels would apply to the calculation of future returns only and would be implemented with a 2-day delay to prevent traders gaming returns based on current large market swings or locked-limit markets. To change NAS level: 1) Go to My Account tab; 2) Click Edit icon—middle icon on right of account; 3) click Change Account Details; 4) Enter new NAS level in Nominal Account Size box.

BASIC STATISTICS.

1. **Cumulative Return** is the aggregate amount that an investment has gained or lost over time, independent of the period of time involved. The cumulative return is a compounded return calculated as follows:

   \[
   Cumulative\ Return = \prod_{t=0}^{T} (1 + R_t) - 1
   \]

   where \( R_t \) is the return on day \( t \) and \( T \) is the number of days in history.

2. **Annualized Return** is the average annual compounded return and given by the formula:

   \[
   Annualized\ Return = \prod_{t=0}^{T} (1 + R_t) \frac{252}{T} - 1
   \]

   where \( R_t \) is the return on day \( t \) and \( T \) is the number of days in history.
3. **Annualized Volatility** is the annualized standard deviation of daily returns and given by the formula:

\[
\text{Annualized Volatility} = \sqrt{252} \cdot sd(R)
\]

where \(sd(R)\) = standard deviation of daily returns. The standard deviation is a measure of the variability in the returns and given by the formula:

\[
sd(R) = \sqrt{\frac{\sum_{t=0}^{T}(R_t - \bar{R})^2}{T - 1}}
\]

where \(R_t\) is the return on day \(t\), \(T\) is the number of days in history, and \(\bar{R}\) is the average return.

4. **Sharpe Ratio** (\(rf=0\))—We define the Sharpe ratio as the annualized compounded daily return divided by the annualized standard deviation (sd) of daily returns as given by the following formula:

\[
\text{Sharpe Ratio} = \frac{\left[\prod_{t=0}^{T}(1 + R_t)\right]^{\frac{252}{T}} - 1}{\sqrt{252} \cdot sd(R)}
\]

where \(R_t\) is equal to the return on day \(t\). (Annualizing the standard deviation requires multiplying by the \(\sqrt{252}\), which is the approximate number of trading days in the year.) This definition differs from the standard definition of the Sharpe ratio in the following ways:

- The standard definition is based on excess returns instead of returns, where excess return is the difference between the return and the risk-free return (typically defined as the T-bill rate). Our calculation is based on a risk-free rate of zero (i.e., \(rf = 0\)). Using a risk-free rate of zero makes the Sharpe ratio invariant to leverage. The standard Sharpe ratio has the characteristic that increasing leverage can increase the ratio, which is an undesirable quality. For example, consider a futures trader who by virtue of trading margined assets can increase exposure without incurring borrowing costs. If the trader's annualized return and standard deviation are both 10%, and the risk-free return is 5%, the Sharpe ratio would be 0.5. If the same trader doubled his exposure, thereby doubling both the return and the standard deviation (but leaving the risk-free return unchanged), the Sharpe ratio would increase to 0.75, even though all the trades are the same except for a uniform doubling in size. If, however, the risk-free return is defined as zero, then the Sharpe ratio is the same (i.e., 1.0) before and after increasing exposure.

- The standard Sharpe ratio uses the arithmetic average return, whereas our formula uses a compounded return. The compounded return over any period of time will exactly match the actual return, whereas the aggregated arithmetic return will not.

- Our Sharpe ratio calculation is based on daily returns, whereas monthly return data is more commonly used. Of course, daily data provides greater statistical significance.

5. **Sortino Ratio**—We divide the Sortino ratio by the square root of 2 for reasons explained below. The Sortino Ratio is a variation of the Sharpe ratio that uses only downside deviation to measure risk instead of the standard deviation, which is based on all returns. The downside deviation is defined as:

\[
\text{dd}(R) = \frac{\sum_{t=0}^{T}(\min(R_t - MAR, 0))^2}{T}
\]

where \(MAR\) = Minimum Acceptable Return and \(T\) = number of days.
A common choice for the MAR is zero, so that the downside deviation calculation is based only on negative returns. (MIN (Rt, 0) is the mathematical definition of a negative return.) The Sortino ratio is therefore equal to the annualized compounded return divided by the annualized standard deviation of all losses (where deviations are measured from zero, not the average loss). Thus, assuming daily data, which requires using an annualizing factor of $\sqrt{252}$, the Sortino ratio is then defined as:

$$\text{Sortino Ratio} = \frac{\left[\sum_{t=0}^{T}(1 + R_t)\right]^{\frac{1}{252}}}{\sqrt{252} \cdot dd(R)}$$

Note that because the Sortino ratio has the same numerator as the Sharpe ratio but calculates the denominator based on the squared deviations of only losing returns, instead of all returns, it will be biased to be higher than the Sharpe ratio, even for traders whose returns are negatively skewed (i.e., large losses are greater in absolute magnitude than large gains). A common mistake is to assume if that if the Sortino ratio is higher than the Sharpe ratio, it implies returns are positively skewed (i.e., large gains are greater in magnitude than large losses). Since the loss measure in the Sortino ratio will be based on summing a smaller number of deviations (i.e., only the deviations of losing returns), the Sortino ratio will almost invariably be higher than the Sharpe ratio. To allow for comparing the Sortino ratio to the Sharpe ratio, we multiply the risk measure of the Sortino ratio by the square root of 2 (which is the same as dividing the Sortino ratio by the square root of 2). Multiplying the risk measure of the Sortino ratio by the square root of 2 will equalize the risk measures of the Sharpe and Sortino ratios when upside and downside deviations are equal, which seems appropriate. When downside deviations are smaller than upside deviations, the Sortino ratio risk measure will still be smaller (and the Sortino ratio higher), even after multiplying by the square root of 2, which is the distinction the Sortino ratio is designed to capture.

6. **FundSeeder Score**—This measure is a complex proprietary formula that is computed based on the time-series of daily returns. When assessing the FundSeeder score, it is important to know the following five facts:
   - The core component of the score is the Probabilistic Sharpe Ratio developed by David H. Bailey and Marcos Lopez de Prado in “The Sharpe Ratio Efficient Frontier”.
   - The score penalizes accounts with an expected one-year-forward maximum drawdown of more than 15%.
   - The score penalizes return distributions with “heavy tails”.
   - The score penalizes traders whose most recent performance is inconsistent with their total performance.
   - All things being equal, traders with longer trading history will get better scores than those with shorter history.

7. The **Maximum Drawdown** measures the largest percentage drop from a relative peak to a subsequent relative low in the value of a portfolio. Define NAV$_i$ = Net Asset Value on day $i$ (equal to either NLV$_i$ or NAS$_i$, depending on which measure is selected as base for calculating returns). The Maximum Drawdown (MD) is then calculated by the following formula:

$$MD = 1 - \min_{j > i} \left(\frac{\text{NAV}_j}{\text{NAV}_i}\right)$$

8. **Daily Gain to Pain Ratio (GPR)**—This ratio is equal to the sum of all daily returns divided by the absolute value of the sum of all daily losses. The GPR essentially shows the ratio of net returns to the losses incurred in getting those returns. The formula for the Daily GPR is given by:

$$GPR = \frac{\sum_{t=0}^{T} R_t}{|\sum_{t=0}^{T} \min(R_t, 0)|}$$
9. **Monthly Gain to Pain Ratio (GPR)**—This ratio is analogous to the daily GPR except that, as the name implies, monthly returns are used in the calculation instead of daily returns. Note: monthly GPRs will always be much higher than daily GPRs (because negative days in a positive month will increase the denominator of the daily GPR but have no impact on the denominator in the monthly GPR). As a rough rule of thumb, the monthly GPR will be about seven times the daily GPR, although this ratio can vary widely. The important point to keep in mind is that daily and monthly GPRs can never be compared with each other.

10. The **MAR Ratio** is the ratio of the annualized compounded return to the maximum drawdown. Both these terms have been defined above. The MAR Ratio gets its name from the Managed Accounts Report newsletter, which developed this metric.

11. The **CALMAR Ratio** is exactly the same as the MAR Ratio except that the calculation is restricted to the past three years of data.

12. **Value-at-Risk (VAR)** is defined as the daily loss level that is expected to be exceeded on less than 5% of all days and is defined by the formula:

\[
VaR = mean(R) - 1.6448 \cdot sd(R)
\]

*Note that this definition of VAR is based on past return levels and not on past prices of the current portfolio.*

13. **Expected Shortfall** is defined as the average loss level of returns below the VAR. Denote by \( L \) the subset of \( R \) such that \( L = \{ R : R_t < VaR \} \). The expected shortfall is defined by \( ES = mean(L) \). The purpose of the expected shortfall is to highlight return distributions in which the worst 5% of returns contain losses much greater than the VAR. For example, an investment with a VAR of 4% and an expected shortfall of 10% would be far riskier than an investment with a 4% VAR and 5% expected shortfall.

14. The **Daily Mean Return** is the arithmetic average of daily returns.

15. The **Daily Standard Deviation** is the standard deviation of the daily returns. The Daily Standard Deviation annualized (multiplied by \( \sqrt{252} \)) is equal to Annualized Volatility.

16. **Skewness** is a measure of symmetry. If the skewness = 0 then the distribution represented by \( R \) is perfectly symmetric. If the skewness is negative, then the distribution is skewed to the left (i.e., large losses tend to be of larger magnitude than large gains), while if the skew is positive, then the distribution is skewed to the right. The calculation of skewness is given by the following formula:

\[
Skewness = \frac{n \cdot \Sigma_{i=1}^{n}(R_i - \bar{R})^3}{(n-1) \cdot (n-2) \cdot sd(R)^3}
\]

17. **Kurtosis** is a measure of peakedness (or flatness). Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution. The calculation of kurtosis is given by the following formula:

\[
Kurtosis = \frac{n \cdot (n + 1)}{(n-1) \cdot (n-2) \cdot (n-3)} \frac{\Sigma_{i=1}^{n}(R_i - \bar{R})^4}{sd(R)^4} - \frac{3 \cdot (n-1)^2}{(n-2) \cdot (n-3)}
\]